Robustness Analysis for Multistage Adaptive Optimization

with Application to Hydrogen Supply Chain Planning in the Netherlands

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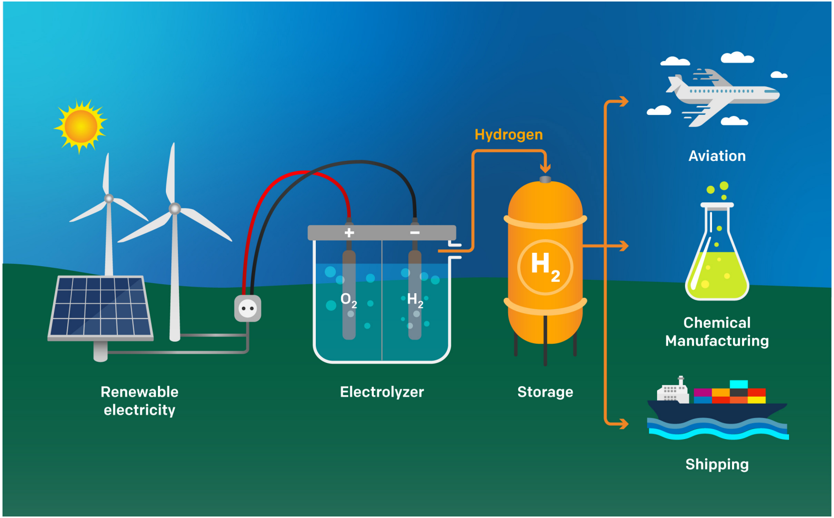
YEEES 31

Outline

Motivation behind the research

- Major concerns regarding current energy system
  - Environmental
  - Geopolitical
  - Economic
- Hydrogen predicted to play an important role in the future

Promise of “green” Hydrogen



Source: Earthjustice (2021)

Main advantages of Hydrogen

1. Can provide energy for “hard to abate” sectors

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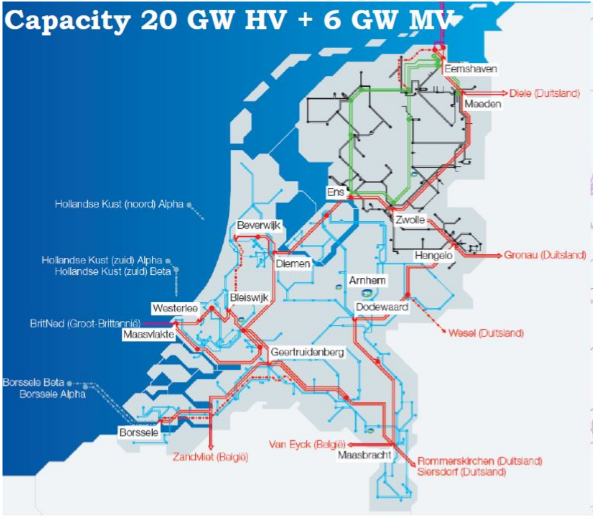
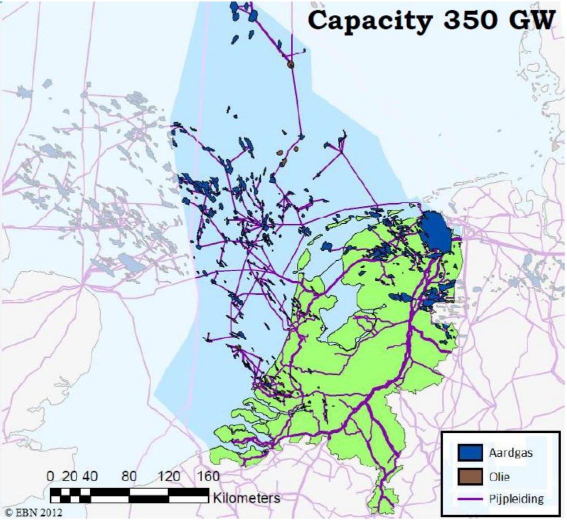
2. Can be transported efficiently at scale

Electricity vs. gas transportation capacity



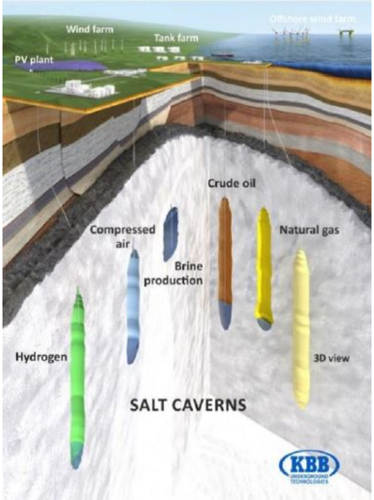


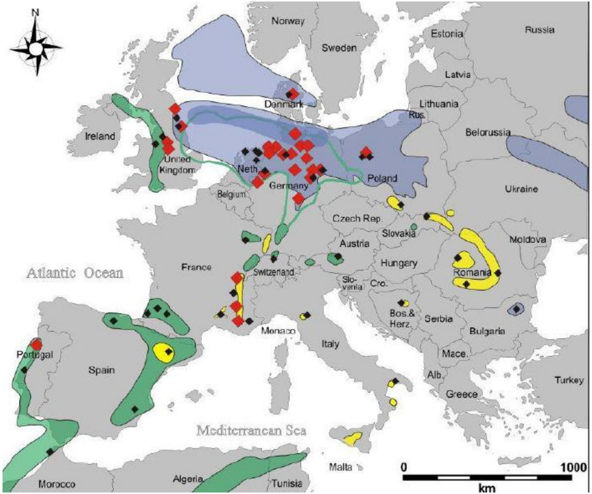
Source: van Wijk (2017)

Main advantages of Hydrogen

1. Can provide energy for “hard to abate” sectors
2. Can be transported efficiently at scale
3. Can be stored efficiently at scale

Hydrogen storage potential





Source: Van Wijk & Wouters (2021)

The Netherlands predicted to play an important role in Europe

- North sea wind generation and salt cavern storage potential
- Strategic location in existing global oil and gas logistics
  - High volume ports
  - Extensive existing gas infrastructure
  - Transportation gateway to North-Western Europe
- Expertise and technology
  - Currently Europe's second largest producer of *fossil-based* hydrogen

However, there are plenty of open questions...

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  - Best production method(s)?
  - Produce locally or import from other countries?

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- When should the relevant investments be made?

$\Rightarrow$ Can use mathematical optimization models to help answer these questions!

Project partners

QuO Mare Quantitative Optimization

TU Delft Delft University of Technology

TNO innovation for life

Kalavasta Climate Neutral Strategies

Dow

FRAMES

STEDIN.NET

YARA

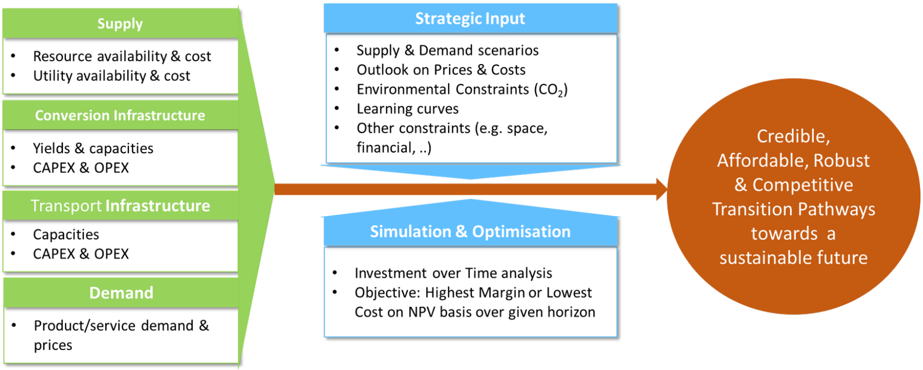
Deltalinqs

PROTON VENTURES

Royal HaskoningDHV Enhancing Society Together

Vopak

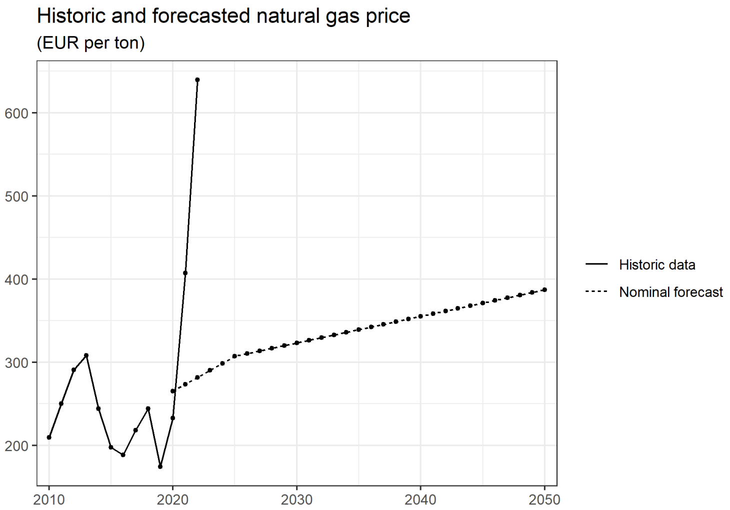
Techno-Economic Optimization Model



- Model hydrogen supply chain over future time horizon $T = \{2020, 2021, \ldots 2050\}$
- Parameters of such a model are highly uncertain

Example: Natural gas price

- One of the most important parameters in the model
- Forecast was made in 2019



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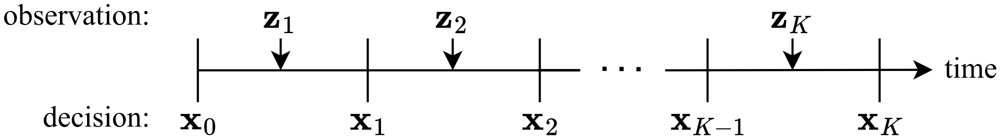
- The parameters of such models are often highly uncertain
- However, some uncertainty is revealed over time and our decisions may adapt accordingly
  - We consider a discrete stage setting
  - Multi-stage adaptive optimization under uncertainty
    - Stochastic Programming (Dantzig (1955))
    - Markov Decision Process (Bellman (1957))
    - Robust Optimization (Ben-Tal & Nemirovski (1999))

Multistage adaptive optimization

- Decisions **x**
- Uncertain parameters **z**



Multistage adaptive optimization

Consider a generic uncertain multistage adaptive optimization problem with K stages:

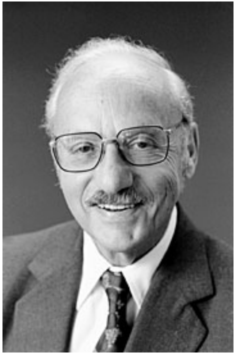
$$\begin{aligned} \min_{\mathbf{x}_0 \in \mathbb{R}^{n_0}} \ & f_0(\mathbf{x}_0) + R_1 \\ \text{s.t. } & \mathbf{x}_0 \in \mathcal{X}_0. \end{aligned}$$

Here R_k represents the "recourse value" at stage k, defined recursively as follows:

$$\begin{aligned} R_k = \min_{\mathbf{x}_k \in \mathbb{R}^{n_k}} \ & f_k(\mathbf{x}_k, \mathbf{z}_{[k]}) + R_{k+1} \\ \text{s.t. } & \mathbf{x}_k \in \mathcal{X}_k(\mathbf{x}_0, \dots, \mathbf{x}_{k-1}, \mathbf{z}_{[k]}), \\ & \vdots \\ R_K = \min_{\mathbf{x}_K \in \mathbb{R}^{n_K}} \ & f_K(\mathbf{x}_K, \mathbf{z}_{[K]}) \\ \text{s.t. } & \mathbf{x}_K \in \mathcal{X}_K(\mathbf{x}_0, \dots, \mathbf{x}_{K-1}, \mathbf{z}_{[K]}). \end{aligned}$$

Remains an open problem . . .

...it is interesting to note that **the original problem that started my research is still outstanding** *- namely the problem of planning or scheduling dynamically over time, particularly* **planning dynamically under uncertainty**. *If such a problem could be successfully solved it could eventually through better planning contribute to the well-being and stability of the world.* - George Dantzig



Standard approach: ignore uncertainty

1. Estimate the uncertain parameters $\boldsymbol{\xi}$ by nominal values $\hat{\boldsymbol{\xi}}$
2. Solve the following deterministic single-level optimization model:

$$\begin{aligned} \min_{\mathbf{x}\in\mathbb{R}^n} \; & f(\mathbf{x}, \hat{\boldsymbol{\xi}}) \\ \text{s.t. } & \mathbf{x} \in \mathcal{X}(\hat{\boldsymbol{\xi}}), \end{aligned}$$

where $n = \sum_{t=1}^{K} n_k$, $f(\mathbf{x}, \hat{\boldsymbol{\xi}}) = \sum_{t=1}^{K} f_k(\mathbf{x}_k, \hat{\boldsymbol{\xi}})$ and $\mathcal{X}(\hat{\boldsymbol{\xi}}) = \bigcap_{k=1}^{K} \mathcal{X}_k(\mathbf{x}_{[k-1]}, \hat{\boldsymbol{\xi}})$.

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- Reduces the problem complexity significantly!
- May lead to a decent "nominal" solution $\implies$ no need to make model more complicated
    - Would like to know whether this is the case ...

Robustness Analysis

Robustness Analysis

Question we would like to answer:

- How robust is a solution?

Contributions of paper:

- Argue why "robustness analysis" can provide valuable insight
- Highlight flaw in widespread use of sensitivity analysis
- Extend methodology of robustness analysis to multistage adaptive setting
- Demonstrate application to hydrogen supply chain planning in the Netherlands

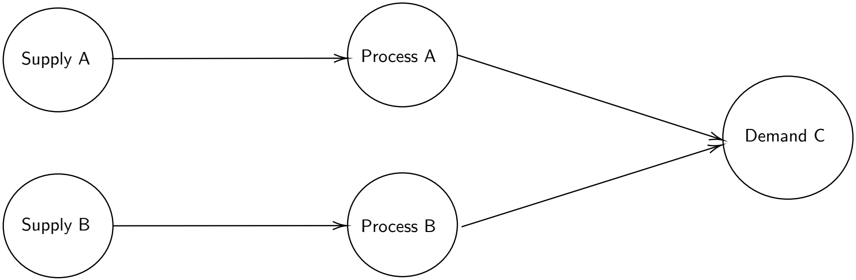
Consider simple toy problem

- 3 Products $p \in \{A, B, C\}$
  - Product C can be created using A or B

- 2 Time periods $t \in \{1, 2\}$

- Objective: satisfy demand of C with minimum costs

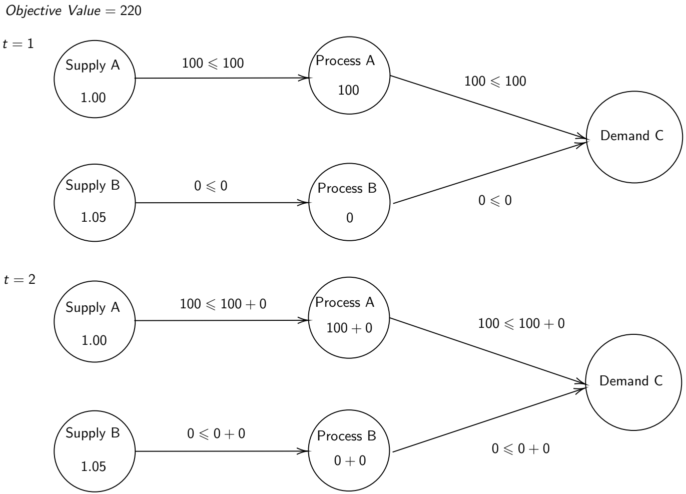
- 5 Nodes
  - 2 Supply
  - 2 Process
  - 1 Demand



Parameters

- Need to produce 100 units of C (in both time periods)
- Supply costs: product A is **cheaper** (on average), but **more volatile**
  - Nominal: $$\begin{pmatrix} \bar{c}_A^1 \\ \bar{c}_A^2 \\ \bar{c}_B^1 \\ \bar{c}_B^2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1.05 \\ 1.05 \end{pmatrix}$$
  - True: $c_A^1 \sim \mathcal{U}(0.5, 1.5)$ and $c_A^2 \sim \mathcal{U}(0.5c_A^1, 1.5c_A^1)$
- Investment costs:
  - Arc capacity increase of 20 units costs 2
  - Process capacity increase of 20 units costs 2

Standard approach (optimize for nominal case) → solution



- Is this a good solution? What if the supply cost of A differs from expectation?

Sensitivity Analysis (SA)

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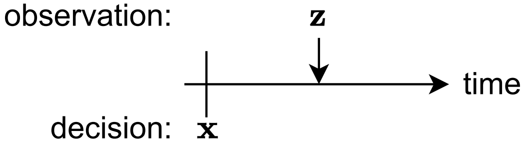
- Robustness analysis relaxes these standard assumptions

Robustness Analysis (RA) in static optimization setting

- Given a fixed solution **x**, what happens if the parameters **z** deviate from the nominal case?

1. Will the solution remain feasible?
2. How might the objective value differ?



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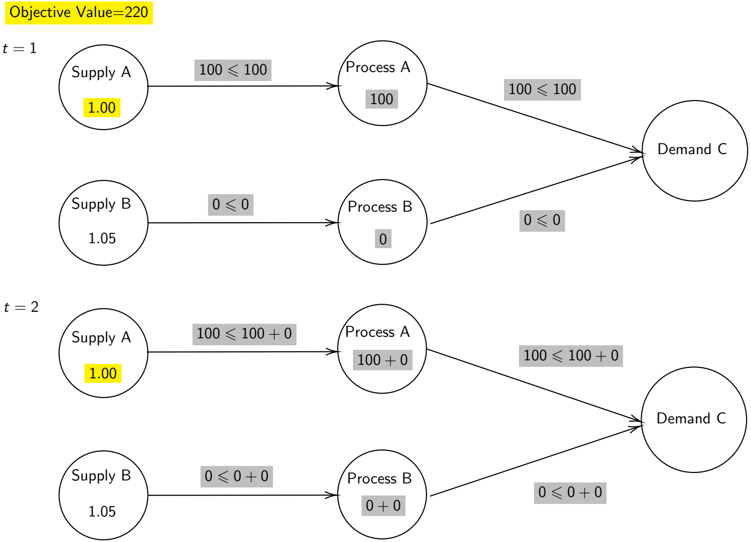
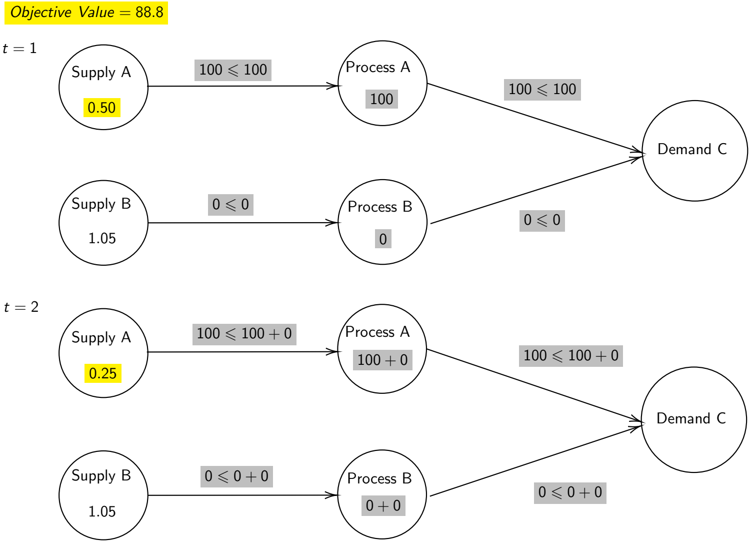


Recall our setup

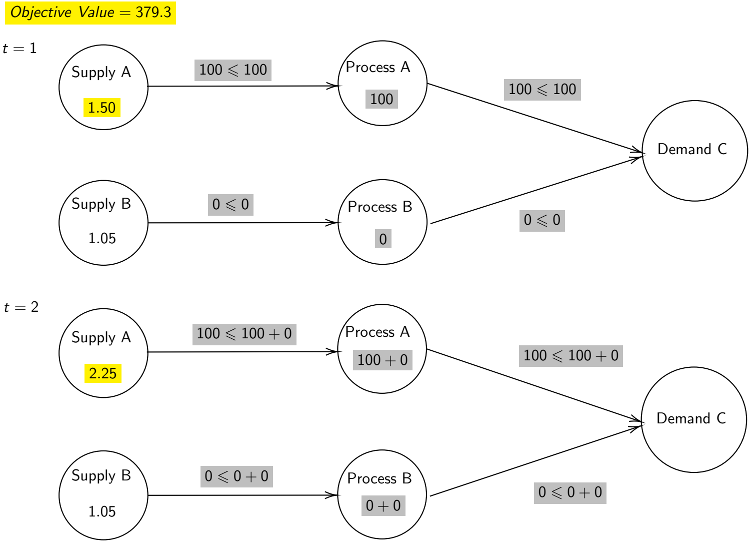
Supply costs: product A is **cheaper** (on average), but **more volatile**

- Nominal: $$\begin{pmatrix} \bar{c}_A^1 \\ \bar{c}_A^2 \\ \bar{c}_B^1 \\ \bar{c}_B^2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1.05 \\ 1.05 \end{pmatrix}$$
- True: $c_A^1 \sim \mathcal{U}(0.5, 1.5)$ and $c_A^2 \sim \mathcal{U}(0.5c_A^1, 1.5c_A^1)$
  - Best case: $c_A^1 = 0.5$ and $c_A^2 = 0.25$
  - Worst case: $c_A^1 = 1.5$ and $c_A^2 = 2.25$

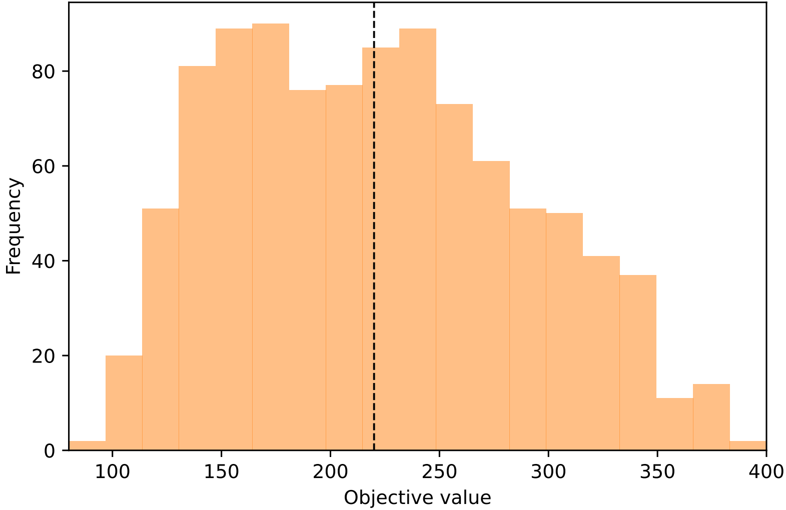
Best case scenario



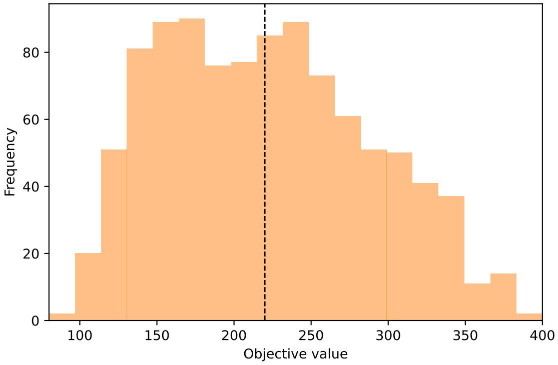
Worst case scenario



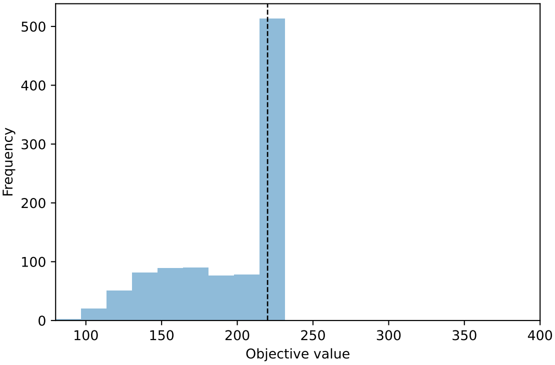
Evaluated on 1000 randomly generated scenarios



Difference RA and SA (in static setting)



(a) Robustness analysis in static setting



(b) Sensitivity analysis

RA assumes the solution is fixed. SA allows the solution to change (with perfect foresight).

Robustness Analysis in adaptive setting

- In reality, not all decisions are set in stone
  - Not fair to consider all variables fixed
  - Some variables are able to adapt to the scenario at hand

Robustness Analysis in adaptive setting

- In reality, not all decisions are set in stone
  - Not fair to consider all variables fixed
  - Some variables are able to adapt to the scenario at hand
- Analysis requires additional component: adaptive decision policy (θ)

Example of an adaptive decision policy θ

$\bar{\theta}$: Folding horizon re-optimization using expectations over future

- In stage t, where $1 \leq t \leq T$, we know $\mathbf{c}_1, \ldots, \mathbf{c}_t$ with certainty and previous decisions $\bar{\mathbf{x}}_1, \ldots \bar{\mathbf{x}}_{t-1}$ are fixed
- Form expectations over future $\hat{\mathbf{c}}_{t+j} = \mathbb{E}[\mathbf{c}_{t+j} | \mathbf{c}_1, \ldots, \mathbf{c}_t], \quad j = 1, \ldots, T-t$
- Determine $\mathbf{x}_t, \ldots, \mathbf{x}_T$ by re-solving model with parameters $\hat{\mathbf{c}} = (\mathbf{c}_1, \ldots, \mathbf{c}_t, \hat{\mathbf{c}}_{t+1}, \ldots, \hat{\mathbf{c}}_T)$ and fixed $\bar{\mathbf{x}}_1, \ldots \bar{\mathbf{x}}_{t-1}$
- Fix $\bar{\mathbf{x}}_t = \mathbf{x}_t$
- $t \leftarrow t + 1$

Applied to our toy problem

- Recall our setup...

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  - Static and adaptive investment variables $\mathbf{x}^1$ and $\mathbf{x}^2(\theta, \mathbf{c}^1)$
    - Arc capacity
    - Processing capacity

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  - Static and adaptive investment variables $\mathbf{x}^1$ and $\mathbf{x}^2(\theta, \mathbf{c}^1)$
    - Arc capacity
    - Processing capacity
  - Adaptive arc flow variables $\mathbf{y}^1(\theta, \mathbf{c}^1)$ and $\mathbf{y}^2(\theta, \mathbf{c}^1, \mathbf{c}^2)$

Applied to our toy problem

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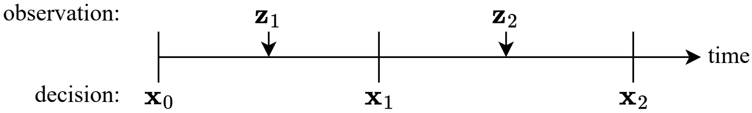
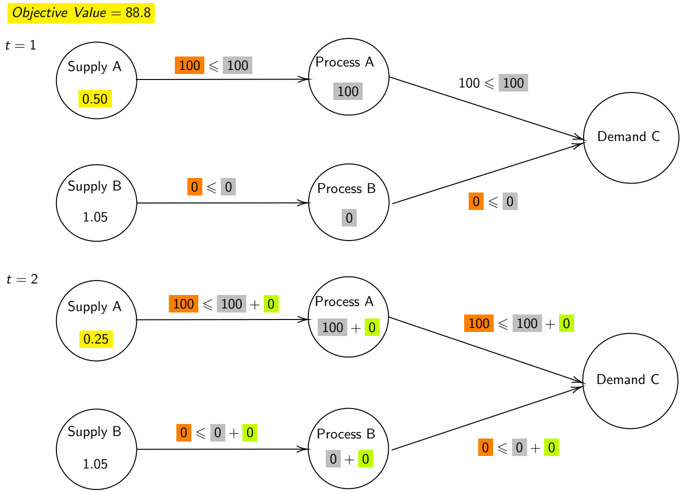
$\Longrightarrow$ Three-stage problem

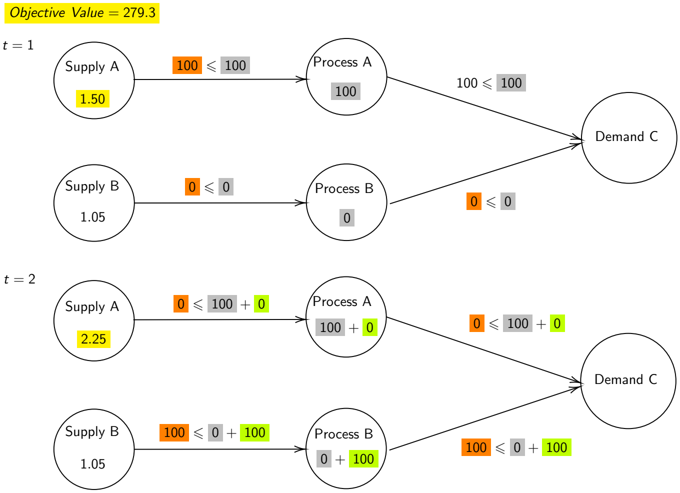


Best case scenario (with adaptive decision policy $\bar{\theta}$)



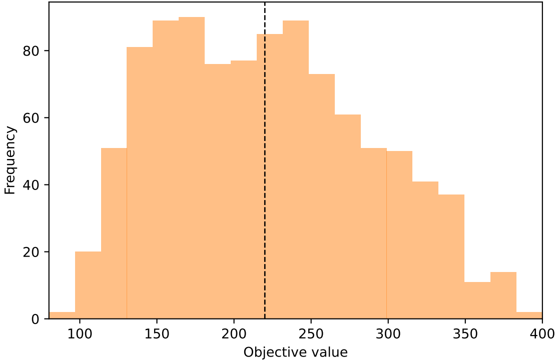
Expectation $\hat{c}_2 = \mathbb{E}[c_2|c_1 = 0.50] = 0.50 \Rightarrow$ no additional investments, happy to stick with product A

“Worst” case scenario (with adaptive decision policy $\bar{\theta}$)

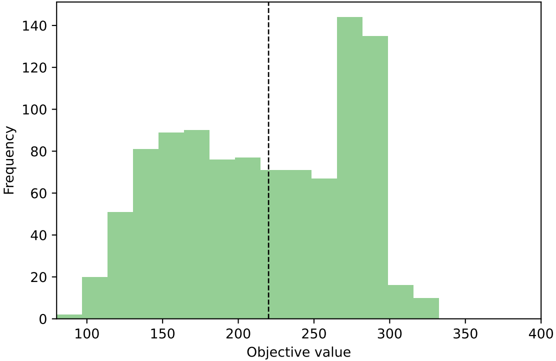


Expectation $\hat{c}_2 = \mathbb{E}[c_2|c_1 = 1.50] = 1.50 \Rightarrow$ decide to make additional investments in product B

Static vs. Adaptive



(a) Robustness analysis in static setting



(b) Robustness analysis in 3-stage adaptive setting

So is the solution sufficiently robust?

- Up to the modeler to decide
- Dependent on situation
- Various risk measures one might want to evaluate
  - $\mathbb{P}$(objective value $\leq$ some threshold)
  - $\mathbb{E}$(objective value)
  - Worst case objective value
  - (Conditonal) Value at Risk
  - ...

Evaluation of risk measures

	$\mathbb{P}(\text{cost} \geq 210.2)$	$\mathbb{E}(\text{cost})$	Worst case	CVaR (10%)
SA	47%	188.0	220.0	220
RA (static)	49%	210.5	375.3	326.6
RA (adaptive)	49%	204.3	302.2	276.3

- SA too optimistic
- RA (static) too pessimistic
- RA (adaptive) provides most realistic assessment

Main takeaways

1. While optimization under uncertainty can be difficult, a posteriori evaluation of a given solution is relatively easy

2. Robustness analysis can be used to assess whether a solution is "sufficiently robust" to parametric uncertainty

3. When modeling an uncertain & adaptive problem setting, our analysis should not be overly optimistic (SA), nor overly pessimistic (static RA)

Thanks for listening! Any questions?

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