Robustness Analysis in a Multistage Adaptive Optimization Setting

With Application to Industrial Decarbonization in the Netherlands

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Parametric Uncertainty

Optimization problem with uncertain input parameter $\tilde{\mathbf{z}}$

$$\min_{\mathbf{x}} \; f(\mathbf{x}, \tilde{\mathbf{z}})$$
$$\text{s.t. } \mathbf{x} \in \mathcal{X}(\tilde{\mathbf{z}})$$

Approaches for dealing with parametric uncertainty:

- Stochastic Programming
- Robust Optimization

⇒ Can be difficult to apply to large-scale problems
⇒ Not always necessary, start with robustness analysis

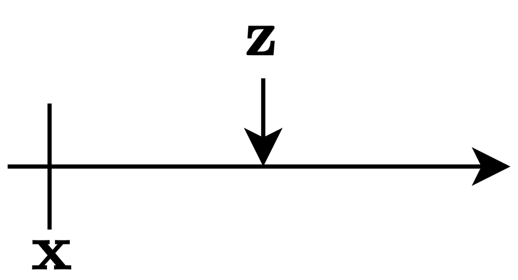
Sensitivity Analysis vs. Robustness Analysis

Under variation in the uncertain input parameter $\tilde{\mathbf{z}}$ . . .

- SA analyzes how the **optimal** solution **changes**
- RA analyzes how a **fixed** solution **performs**

- SA commonly assumes that:
  (i) All decisions are adaptable
  (ii) Decisions are able to adapt with perfect foresight
- RA discards assumptions (i) and (ii)

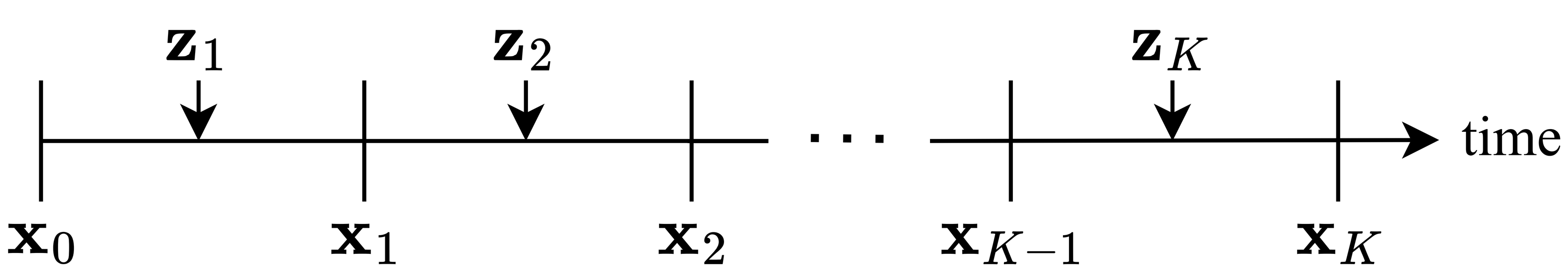
Static Setting



Input: Set of scenarios $\{\mathbf{z}^1, \ldots, \mathbf{z}^N\}$. Given solution $\bar{\mathbf{x}}$.
Output: Evaluation of solution $\bar{\mathbf{x}}$ w.r.t. each scenario in $\mathcal{S}$.

1: **for** scenario $i \in \{1, \ldots, N\}$ **do**
2: Evaluate feasibility by determining whether $\bar{\mathbf{x}} \in \mathcal{X}(\mathbf{z}^i)$
3: Evaluate objective value by computing $f(\bar{\mathbf{x}}, \mathbf{z}^i)$

Multistage Adaptive Setting



Input: Set of scenarios $\{\mathbf{z}^1, \ldots, \mathbf{z}^N\}$. Given static here-and-now decisions $\bar{\mathbf{x}}_0$ and adaptive decision policy $\bar{\theta}$.
Output: Evaluation of solution $(\bar{\mathbf{x}}_0, \bar{\theta})$ w.r.t. each scenario in $\mathcal{S}$.

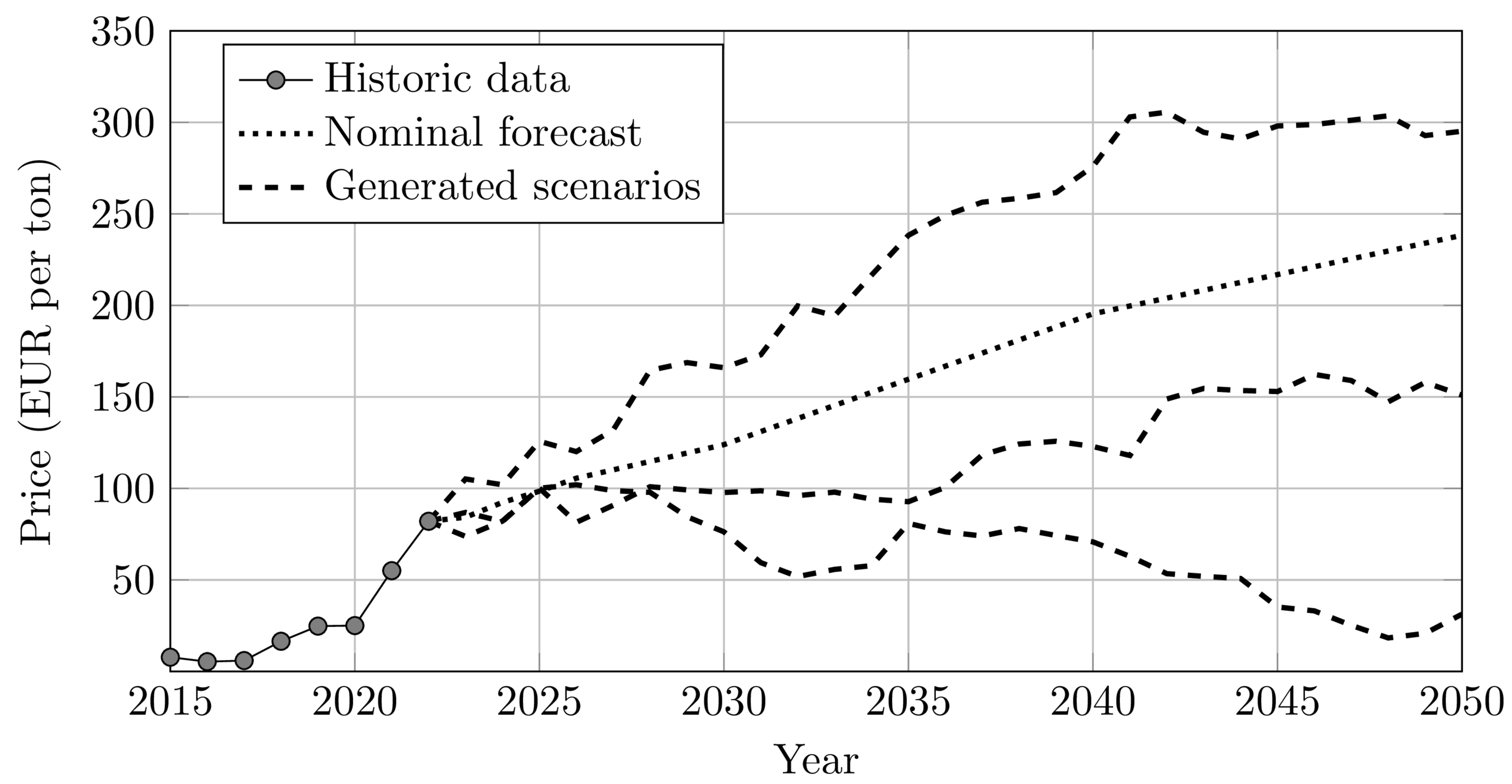
1: Fix first-stage decisions $\bar{\mathbf{x}}_0$
2: **for** scenario $i \in \{1, \ldots, N\}$ **do**
3: **for** stage $k \in \{1, \ldots, K\}$ **do**
4: $\mathbf{z}^i_k$ is observed
5: Implement $\bar{\theta}$ to determine $\bar{\mathbf{x}}_k$
6: $\bar{\mathbf{x}} \leftarrow (\bar{\mathbf{x}}_0, \bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_K)$
7: Evaluate feasibility by determining whether $\bar{\mathbf{x}} \in \mathcal{X}(\mathbf{z}^i)$
8: Evaluate optimality by computing $f(\bar{\mathbf{x}}, \mathbf{z}^i)$

Scientific Contributions

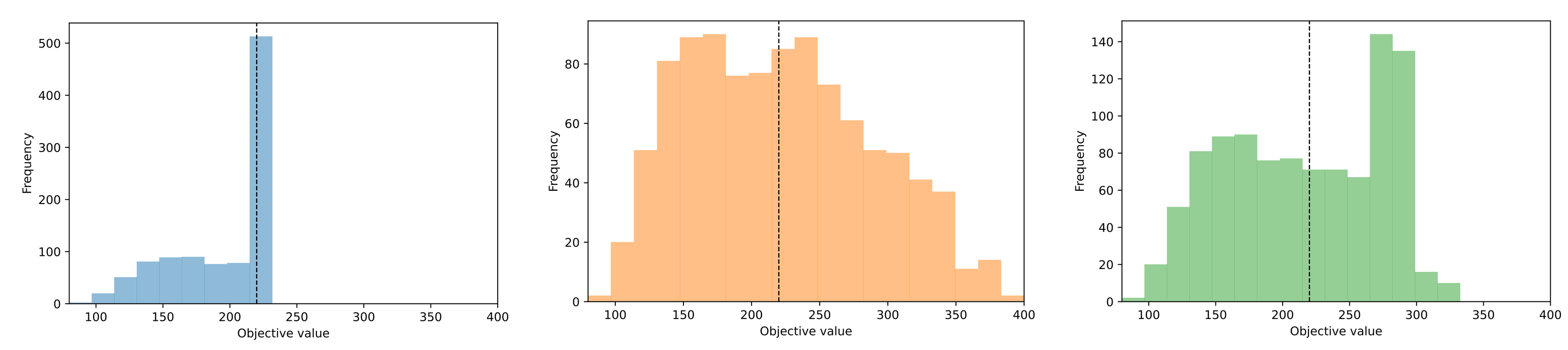
- Highlight difference between SA and RA
- Argue that, in many situations, RA has more practical value than SA
- Extend RA methodology to a multistage adaptive setting
- Demonstrate RA methodology with realistic application

Application

- Goal: optimize decarbonization pathway (2020-2050) for an industrial cluster in the Netherlands
- Possible via a combination of:
  - Carbon capture and storage
  - Electrification
  - Hydrogen usage
- MILP model with many uncertain parameters
  - Example: EU ETS carbon price



- Different methodology leads to different results
  - Comparison when applied to an illustrative toy problem. The dotted line indicates the nominal/expected objective value.



(a) SA (b) RA (static) (c) RA (adaptive)

- SA is too optimistic
- RA in a static setting is too pessimistic
- RA in an adaptive setting is the most realistic assessment!

Project Partners

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